Aero-Optic Wavefront Characterization via Optimized Dynamic Mode Decomposition

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Abstract

- Aero-optical beam control relies developing low-latency forecasting techniques to quickly predict wavefronts aberrated by the turbulent boundary layer (TBL) around an airborne optical system.
- We leverage the optimized DMD (opt-DMD) algorithm on a subset of the Airborne Aero-Optics Laboratory-Transonic (AAOL-T) experimental dataset, characterizing aberrated wavefront dynamics over a hemispherical laser turret for 23 beam propagation directions.
- We show that opt-DMD produces an optimally debiased eigenvalue spectrum with imaginary eigenvalues, allowing for arbitrarily long forecasting to produce a robust future state prediction, while exact DMD loses structural information due to modal decay rates.

Background on Aero-optics

- For an airborne optical platform, the adaptive optic (AO) system must correct wavefront distortions from three primary sources: mechanical jitter, atmospheric effects, and near-field effects where the TBL rapidly alters the refractive index - the focus of this work.
- The index of refraction, n, is directly linked to air density fluctuations by

$$n(\mathbf{r}) = 1 + K_{GD}(\lambda_0)\rho(\mathbf{r}), \tag{1}$$

  where K_{GD} is the wavelength-dependent Gladstone-Dale factor, λ_0 is the laser wavelength, and $\rho(\mathbf{r})$ is the air density as a function of position $\mathbf{r}$.
- Characterizing propagating beam wavefront dynamics in the TBL is critical to correcting the outgoing phase profile of the beam.
- We quantify the aero-optic wavefront aberrations by the calculating optical path difference (OPD) of the beam and normalizing by the Mach number as a ratio of the speed of sound M, turret diameter D, and ratio of in-flight to sea-level air density ρ/ρ_0.

Experimental In-flight AAOL-T Data

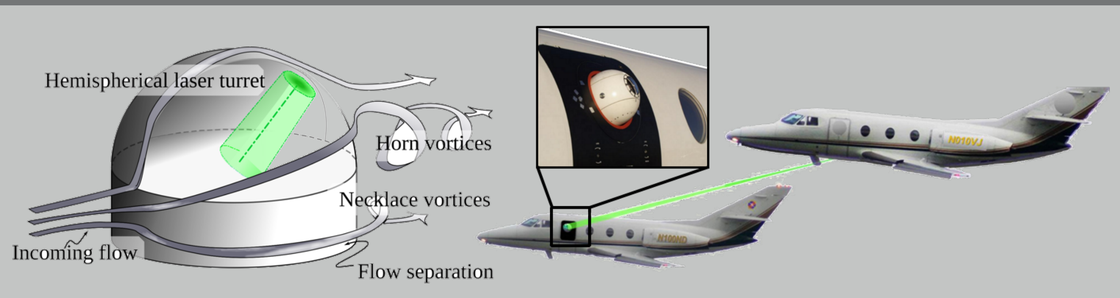

Figure 1:Diagram of the flow past the hemispherical turret aboard the AAOL-T aircraft.

The AAOL-T from University of Notre Dame records live aero-optical data in flight using a 532-nm source beam propagating from a hemispherical laser turret mounted on a Falcon 10 aircraft, as in Figs. 1 and 2. The beam overfills the pupil aperture on the receiver laboratory aircraft. A Shack–Hartmann wavefront sensor (SHWFS), shown in Fig. 3, is used to capture wavefront phase aberrations between the source and destination beams.

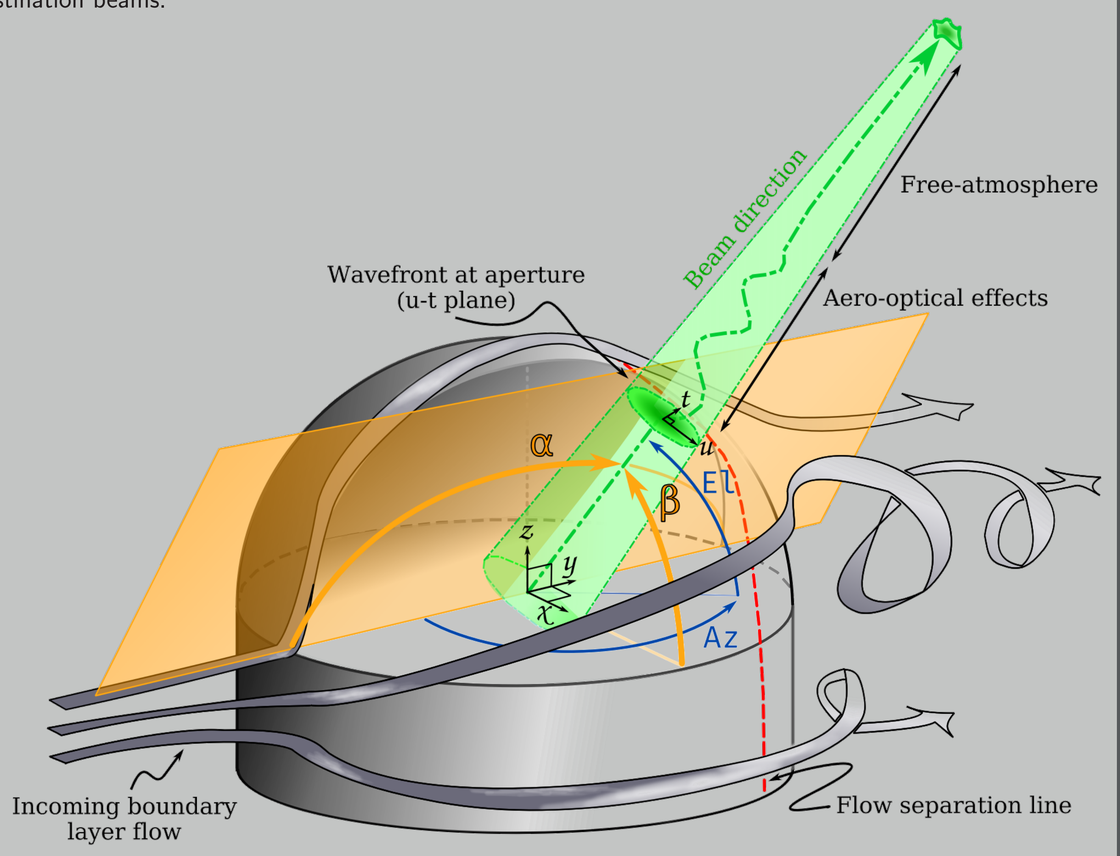

Figure 2:Detail of the turret geometry. The aperture (green disc) images the aberrated wavefronts. Wavefront distortion is highly dependent on beam direction, parametrized by α and β.

Beam direction is analyzed in terms of its "look-back" and inclination angles, α and β respectively. A look-back angle of α =0° implies the beam propagates in the forward direction of the aircraft, while 180° would point to the rear into the turbulent flow separation region. Similarly, β =0° would point towards the ground, while β =180° would be skyward, assuming the aircraft is flying parallel to the surface.

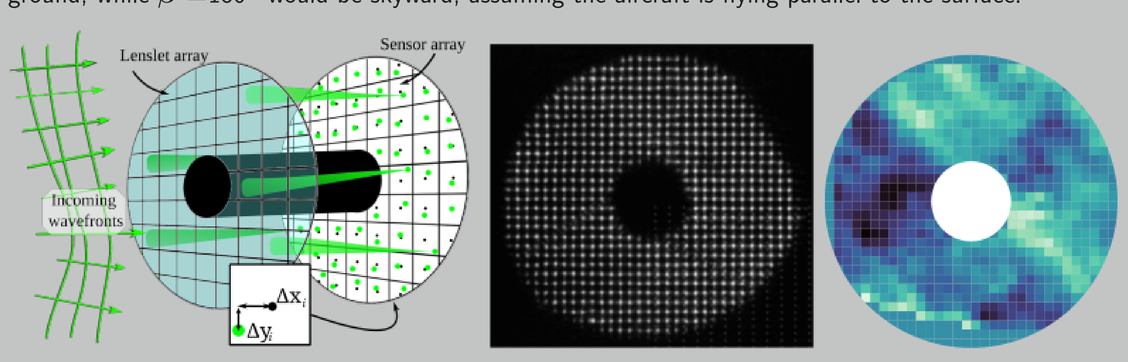

Figure 3:Geometry of the SHWFS on the AAOL-T laser turret with an incident aberrated wavefront alongside the unprocessed and processed data showing intensities on the 32 x 32 subaperature sensors. The lenslet arrays project to the sensor array where the displacements from the sensor centroids are used to compute the local tilts of the wavefront for reconstruction.

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Aero-optic Effect as a Function of Look-back Angle.

The OPD, as determined by SHWFS measurements, becomes our indicator of wavefront aberrations - and as a result - of turbulent activity around the laser apparatus. The effects of propogation angles α and β is visualized in Fig. 3.

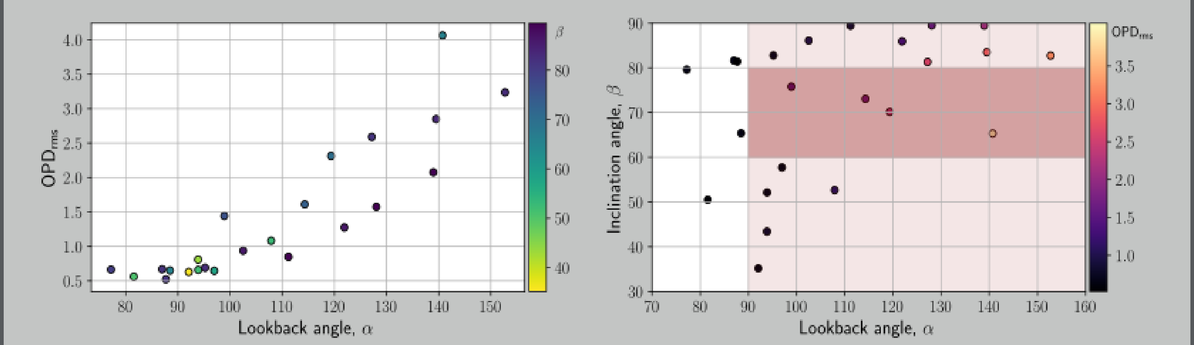

Figure 4:Beam propagation direction affects OPD, as shown in the AAOL-T data, where increasing look-back angle is correlated with greater OPD, and therefore, aberrated wavefronts.

Exact DMD and opt-DMD Algorithms

The DMD framework, developed by Schmid and furthered by Tu et al, seeks the leading spectral decomposition of the best-fit linear operator, $\mathbf{A}$, which acts as a flow map relating two snapshot matrices in time by

$$\mathbf{X}' \approx \mathbf{A}\mathbf{X}. \tag{2}$$

The best-fit operator, $\mathbf{A}$, then establishes a linear dynamical system that best advances snapshot measurements forward in time. If we assume uniform sampling in time, this becomes

$$\mathbf{x}_{k+1} \approx \mathbf{A}\mathbf{x}_k. \tag{3}$$

The snapshot matrices themselves are composed of state vectors shifted in time,

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}(t_1) & \mathbf{x}(t_2) & \cdots & \mathbf{x}(t_m) \\ | & | & & | \end{bmatrix} \qquad \mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}(t_2) & \mathbf{x}(t_3) & \cdots & \mathbf{x}(t_{m+1}) \\ | & | & & | \end{bmatrix}. \tag{4a}$$

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The opt-DMD algorithm of Askham and Kutz uses a variable projection method for nonlinear least squares, providing the best performance of any algorithm currently available. In opt-DMD, the data matrix $\mathbf{X}$ may be reconstructed as

$$\mathbf{X} \approx \underbrace{\begin{bmatrix} | & & | \\ \phi_1 & \cdots & \phi_r \\ | & & | \end{bmatrix}}_{\mathbf{\Phi}} \underbrace{\begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_r \end{bmatrix}}_{\mathrm{diag}(\mathbf{b})} \underbrace{\begin{bmatrix} e^{\lambda_1 t_1} & \cdots & e^{\lambda_1 t_m} \\ \vdots & \ddots & \vdots \\ e^{\lambda_r t_1} & \cdots & e^{\lambda_r t_m} \end{bmatrix}}_{\mathbf{T}(\lambda)}, \tag{5}$$

where the i^{th} eigenmode, ϕ_i, has a corresponding mode amplitude b_i and eigenvalue λ_i. The opt-DMD algorithm directly solves the exponential time dynamics fitting problem,

$$\min_{\lambda, \mathbf{\Phi}_\mathbf{b}} \|\mathbf{X} - \mathbf{\Phi}_\mathbf{b}\mathbf{T}(\lambda)\|_F, \qquad \mathbf{\Phi}_\mathbf{b} = \mathbf{\Phi}\mathrm{diag}(\mathbf{b}). \tag{6}$$

Results

The opt-DMD analysis, results of which are seen in Fig. 5, creates a perfectly debiased spectrum, whose purely imaginary eigenvalues allow for exponential time dynamics with little time-deacy and arbitrarily long forecasts.

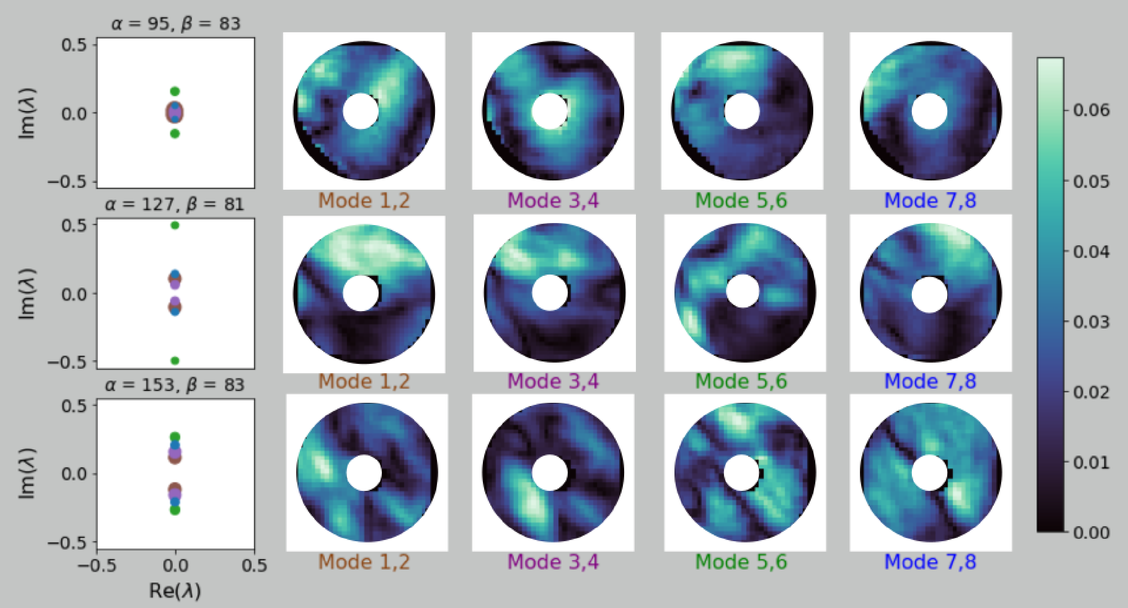

Figure 5:Each row depicts the opt-DMD decomposition for a (α, β) beam direction including the eigenvalues and first eight modes (not picturing complex-conjugates).

In contrast, exact DMD eigenvalues such as those in Fig. 6, result in eigenvalues with a real component. Therefore the modal decomposition decays, losing out on arbitrarily long forecasting.

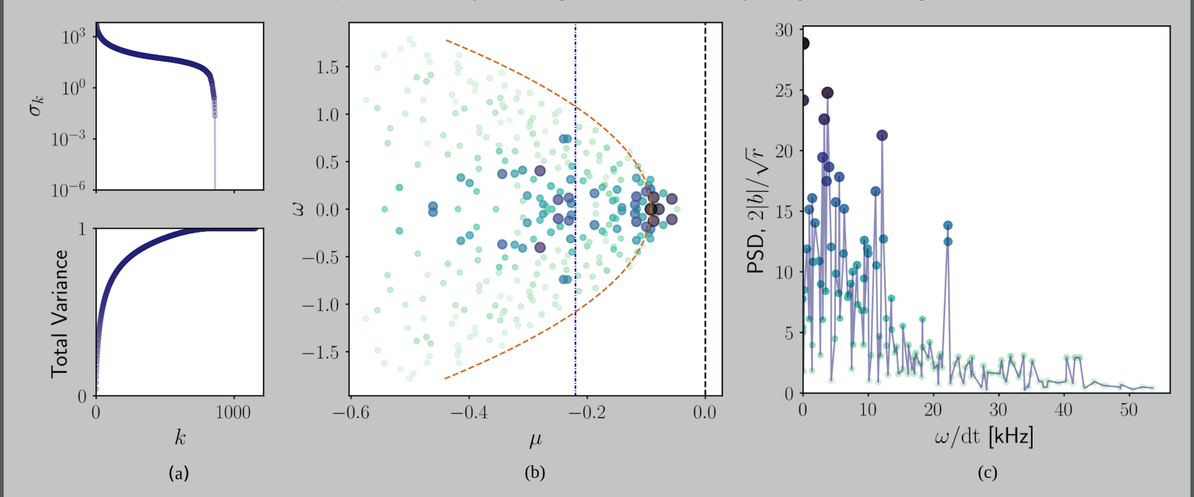

Figure 6:Exact DMD of the α =153°, β = **83**° data seen in third row of Fig 5. Note the negative components of the eigenvalues.

We note here another advantage of opt-DMD that is not used in the current work: opt-DMD does not require discretely spaced temporal snapshots! The flexibility, robustness, and speed of opt-DMD make it an excellent candidate for forecasting aero-optical phenomena.

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